On the Empirical Optimization of Antenna Arrays

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Abstract

Empirical optimization is an algorithm for the optimization of antenna array performance under realistic conditions, accounting for the effects of mutual coupling and scattering between the elements of the array and the nearby environment. The algorithm can synthesize optimum element spacings and optimum element excitations; and is applicable to arrays of various element types having arbitrary configurations, including phased arrays, conformal arrays and non-uniformly spaced arrays. The method is based on measured or calculated element pattern data and proceeds in an iterative fashion to the optimum design. A novel method is presented in which the admittance matrix representing an antenna array consisting of both active and passive elements, is extracted from the array's element pattern data. The admittance matrix formulation incorporated into the empirical optimization algorithm enables optimization of the location of both passive and active elements. The method also provides data for a linear approximation of coupling as a function of (non-uniform) element locations, and for calculation of element scan impedances. Computational and experimental results are presented that demonstrate the rapid convergence and effectiveness of empirical optimization in achieving realistic antenna array performance optimization.

Keywords: antenna arrays; phased arrays; non-uniformly spaced arrays; optimization methods; electromagnetic coupling; admittance matrix; impedance matrix

1. Introduction

TRADITIONALLY, antenna array design has been based on an analytic approach. This has led to many elegant closed form solutions but it has also often led to design methods that are limited by restrictive conditions, such as the need for regularity in the array configuration (i.e. uniform element spacing); and unrealizable assumptions (particularly concerning the effects of mutual coupling). For example, a Chebyshev distribution is optimum only for the idealized case of a uniformly $\lambda/2$ spaced array of isotropic elements with no coupling [1].

The antenna literature contains many useful methods for the optimum synthesis of antenna arrays. These include the use of non-uniform spacing [2,3], methods to account for mutual coupling [4,5,6], and the use of numerical search [7,8,9,10].

Empirical optimization is an algorithm for the optimization of antenna array performance under realistic conditions, accounting for the effects of mutual coupling and scattering between the elements of the array and the nearby environment [11,12]. The method is based on measured or calculated element pattern data and proceeds in an iterative fashion to the optimum design. The algorithm can synthesize optimum element spacings and optimum element excitations; and is applicable to arrays of various element types having arbitrary configurations, including phased arrays, conformal arrays and non-uniformly spaced arrays.

This paper presents two versions of the empirical optimization algorithm. Both versions are applicable to the optimum synthesis of array element excitations. Version v1 can be used for the optimization of active element locations when inter-element spacing is $> 0.5\lambda$ and coupling effects do not vary rapidly as a function of element locations. Version v1 accounts for the presence of passive elements in the near vicinity of the array, but it cannot be used to optimize their locations. This can be done with Version v2 of the algorithm.

Version v2 presents a novel method for the extraction of the admittance matrix representation of an antenna array. The admittance matrix contains the effects of electromagnetic coupling between the active and passive elements of the array. The method presented is applicable to arrays with non-uniform spacing. Other methods to account for coupling effects in arrays described in the literature normally require uniform element spacing. The admittance matrix formulation provides a means to i) account for coupling between the active and passive array elements as a function of their locations, and ii) to calculate the active element scan impedances.

The empirical optimization method can find both the optimum set of array element locations (non-uniform spacing) as well as the optimum set of element excitations. This provides added degrees of freedom in achieving optimum array performance and in compensating for coupling effects, as compared to traditional analytical design methods. Non-uniform spacing offers special advantages in suppressing grating lobes in thinned arrays and in wide angle scan and broad frequency bandwidth array operation.

A numerical function minimization method is used to find the set of array parameters (element excitations and/or element locations) to minimize the normed difference between the actual array pattern and some desired pattern. The use of numerical function minimization methods for optimum search removes the restrictions normally imposed by analytic methods for array geometry regularity. The use of asymmetric excitation distributions and non-uniform element spacing allows for increased degrees of freedom in design. It also allows for the optimization of arbitrary array configurations including conformal arrays. The use of embedded element pattern data means that the optimization is performed under realistic conditions that account for the effects of electromagnetic coupling between the elements of the array and the environment.

Examples of antenna array optimization results obtained by computer simulation are presented in Section 7 of this paper. Experimental results obtained with the empirical optimization algorithm using measured data are shown in references [11,12].

2. Formulation of the Empirical Optimization Algorithm, v1

The design variables that are normally used to synthesize an antenna array are the set of complex valued element excitations, $\mathbf{a} = \{a_n\}$, which are unit-less weights, and the set of element locations in three dimensional space $\mathbf{r} = \{\mathbf{r}_n\}$, where the index n denotes the nth element in the array. The array field pattern, a complex valued function of angle and the design parameters is given by:

$$E(\phi', \mathbf{a}, \mathbf{r}) = \sum_{n=1}^{N} h_n(\phi', \mathbf{r}) a_n e^{j \frac{2\pi}{\lambda} \mathbf{r}_n \cdot \hat{\mathbf{u}}(\phi')} \tag{1}$$

where $h_n(\phi', \mathbf{r})$ is the complex valued field pattern of the nth element measured with all other elements terminated in their characteristic impedance, $\phi' = (\theta, \phi)$ is the observation angle, $\hat{\mathbf{u}}(\phi')$ is the unit vector in the direction of the far-field observation angle and N is the total number of actively excited antenna elements in the array. The element patterns are variously referred to in the literature as the active, embedded or in-situ element patterns [4,11,12]. Each element pattern, $h_n(\phi', \mathbf{r})$, is theoretically a function of the set of element locations, $\mathbf{r}$. The element pattern data includes the effects of coupling between the elements of the array and the nearby environment. It is assumed that the elements operate as unimodal antennas [5,6], and that the element patterns are independent of the element excitations.

The array pattern normalized at the angle ϕ'_0 is given by the conjugate product:

$$P(\phi', \mathbf{a}, \mathbf{r}) = \frac{E(\phi', \mathbf{a}, \mathbf{r}) \tilde{E}(\phi', \mathbf{a}, \mathbf{r})}{E(\phi'_0, \mathbf{a}, \mathbf{r}) \tilde{E}(\phi'_0, \mathbf{a}, \mathbf{r})} \tag{2}$$

where $\tilde{E}$ represents the complex conjugate of the electric field.

In general, not all the element excitations, a_n, and locations, $\mathbf{r}_n$, are variable. For example, one element's excitation amplitude and phase can be fixed, and the outer element locations can be fixed, thereby fixing the maximum physical dimensions of the array. We denote the number of variable array parameters by NV and the vector of variable array parameters by $\mathbf{v} = (v_1, v_2, \ldots, v_{NV})$. The coordinates of $\mathbf{v}$ are a subset of the coordinates of the vector of excitations, $\mathbf{a}$, or the vector of locations, $\mathbf{r}$. The optimization problem can be defined as that of finding the coordinate values of $\mathbf{v}$ that minimize the norm of the difference between the actual array pattern $P(\phi', \mathbf{v})$, and some desired pattern $P_D(\phi')$, i.e., find:

$$\min_{\mathbf{v}} \| P(\phi', \mathbf{v}) - P_D(\phi') \| \tag{3}$$

The choice of norm depends on the specific array performance desired. For minimization of the maximum array pattern sidelobe, the max norm is used, i.e., find:

$$\min_{\mathbf{v}}, \max_{\phi_1' < \phi' < \phi_2'} P(\phi', \mathbf{v}) \tag{4}$$

where $\phi_1' < \phi' < \phi_2'$ defines the sidelobe region in angular space, and $P_D(\phi') = 0$. If it is desired to minimize the power in the sidelobe region, an l_2 norm is used, i.e., find:

$$\min_{\mathbf{v}} \sum_{\phi_1' < \phi' < \phi_2'} P(\phi', \mathbf{v}) \tag{5}$$

For min-max shaped beam synthesis, the criterion statement would be, find:

$$\min_{\mathbf{v}}, \max_{\phi_1' < \phi' < \phi_2'} |P(\phi', \mathbf{v}) - P_D(\phi')| \tag{6}$$

where $\phi_1' < \phi' < \phi_2'$ defines the region in angular space where it is desired to synthesize the pattern $P_D(\phi')$, for example: $P_D(\phi') = \csc^2(\phi')$. It is usual to

1) search for the optimum set of element locations with all element excitations fixed; or
2) search for the optimum set of variable excitations, a_n, with all element locations fixed.

Search procedures 1 and 2 are usually not done simultaneously in recognition of the fact that the element patterns $h_n(\phi', \mathbf{r})$ are dependent on the set of element locations, $\mathbf{r}$, whereas they are independent of the set of element excitations, $\mathbf{a}$. To optimize both element locations and excitations, procedures 1 and 2 may be performed sequentially. Version v1 of the empirical optimization algorithm, incorporating the procedure outlined above for convergence to the optimum set of array parameters, is given in Sidebar 1.

SIDEBAR 1 - THE EMPIRICAL OPTIMIZATION ALGORITHM, V1

1) Set initial array parameter values: $\mathbf{r}^\circ, \mathbf{a}^\circ, \mathbf{v}^\circ$.
2) Measure or compute $P(\phi', \mathbf{v}^\circ)$.
3) Evaluate $f(\mathbf{v}^\circ) = \| P(\phi', \mathbf{v}^\circ) - P_D((\phi') \|$,
   if $f(\mathbf{v}^\circ) \leq \varepsilon$, STOP, $\varepsilon =$ convergence criterion;
   Otherwise continue.
4) Measure or compute the element patterns, $h_n(\phi', \mathbf{r}^\circ), n = 1 \ldots N$.
5) Set

$$E(\phi', \mathbf{v}) = \sum_{n=1}^{N} h_n(\phi', \mathbf{r}^\circ) a_n e^{j\frac{2\pi}{\lambda}\mathbf{r}_n \cdot \hat{\mathbf{u}}(\phi')}$$

   and

$$P(\phi', \mathbf{v}) = \frac{E(\phi', \mathbf{v}) \tilde{E}(\phi', \mathbf{v})}{E(\phi_\circ', \mathbf{v}) \tilde{E}(\phi_\circ', \mathbf{v})}$$

   normalized at angle ϕ_0'.
   (Partial derivatives, $\frac{\partial P(\phi', \mathbf{v})}{\partial v_n}$, can also be obtained if required)
6) Set $f(\mathbf{v}) = \| P(\phi', \mathbf{v}) - P_D(\phi') \|, \mathbf{v}$ variable and using iterative numerical search, find $\min f(\mathbf{v}) \longrightarrow \mathbf{v}^*$.
7) Set $\mathbf{v}^\circ = \mathbf{v}^*$, go to step 2 and repeat.

Note 1: In Step 2, $P(\phi', \mathbf{v}^\circ)$ is obtained either by measurement at the output of the array corporate feed or by computation of the array pattern accounting for coupling effects; without requiring element pattern data.

Note 2: The convergence criterion ϵ, is determined by measurement tolerances and/or performance specifications.
Note 3: In step 5, the element patterns, $h_n(\phi', \mathbf{r}^\circ)$, are obtained for a fixed set of element locations, $\mathbf{r}^\circ$, and are independent of element excitations $\mathbf{a}$. They are not functions of the variable array parameters, $\mathbf{v}$. Therefore the array pattern in step 5 is an analytic function of $\mathbf{v}$, and closed form expressions for partial derivatives with respect to the elements of $\mathbf{v}$ can be readily obtained and used in the function minimization search of step 6.
END Sidebar 1

3. Optimum Search/Function Minimization Method

The function $f(\mathbf{v}) = \|P(\phi', \mathbf{v}) - P_D(\phi')\|$, the normed difference between the array pattern and some desired pattern, defines an $NV + 1$ dimensional surface over the NV-dimensional parameter space, $\mathbf{v}$. Minimization of $f(\mathbf{v})$ optimizes array performance. However, $f(\mathbf{v})$ is a nonlinear function of $\mathbf{v}$. Therefore, minimization requires an iterative numerical search in the space $\mathbf{v}$. The mathematics literature contains many ingenious methods for such non-linear function minimization [13,14]. These methods broadly fall into two classes; direct methods that require only function evaluations; and gradient methods that also require partial derivatives. The Nelder-Mead simplex method [15], is a robust, direct search method that does not require gradient information. Modified gradient search methods generally provide more rapid convergence to an optimum [16].

It is important to note that nonlinear function minimization does not guarantee convergence to a 'global' minimum. Success in nonlinear search is dependent on the initial starting point. Therefore it is extremely important that the initial design parameters, i.e. the values of $\mathbf{r}^\circ, \mathbf{a}^\circ,$ and $\mathbf{v}^\circ$ (Sidebar 1, step 1), be based on valid analytic and physical design principles.

4. Element Excitation Optimization

Optimization of the excitations in a realistic array is necessary to account for the effects of coupling, variably directive element patterns and possibly non-uniform element spacing. For example, a Chebyshev distribution is optimum only for the idealized case of a uniformly spaced array of isotropic elements with no coupling.

The element patterns $h_n(\phi', \mathbf{r})$ are independent of the set of element excitations because the effects of electromagnetic coupling do not change with changes in element excitations. Therefore, the optimum set of element excitations, with all locations fixed, can usually be found in a single iteration of the empirical optimization algorithm.

Element excitation optimization can be effective in controlling near-in sidelobes and compensating for the effects of coupling. The appearance of grating lobes due to array thinning and/or wide angle scan and/or broad frequency bandwidth operation, is an important problem in modern phased array antennas. Optimum non-uniform element spacing is an effective means to suppress grating lobes; whereas, element excitation control has no effect on grating lobes.

5. Element Location Optimization

The active element patterns $h_n(\phi', \mathbf{r})$ are dependent on the element locations, $\mathbf{r}$, due to electromagnetic coupling between the elements of the array and the environment. Non-linear function minimization is an iterative procedure. In the numerical search for the optimum set of element locations (Sidebar 1, step 6), it would be unduly burdensome to require either measurement or computation of a new set of element patterns for each new setting of element locations. Therefore, the numerical search for the optimum set of element locations is based on the set of element patterns that was obtained at the start of the search. At the end of the numerical search, a new set of element patterns must be measured or computed for the new set of element locations. If array performance at the new set of element locations is satisfactory, the process ends. If not, element pattern data is updated and the method proceeds in an iterative fashion.

Algorithm v1, given in Sidebar 1, accounts for the presence of passive elements, but it cannot be used to optimize the locations of the passive elements in the array. To do this, an extended version (v2) of the empirical optimization algorithm is presented in Sidebar 2.

6. Coupling Effects and Optimization as a Function of Element Locations

In this section a method is presented in which the admittance matrix representing an antenna array consisting of both active and passive elements, is extracted from the array's element pattern data. The admittance matrix formulation incorporated into the empirical optimization algorithm enables optimization of the location of both passive and active elements. The method also provides data for a linear approximation of coupling as a function of (non-uniform) element locations, and for calculation of element scan impedances.

6.1 Impedance/Admittance Matrix Formulation

In the following we consider an array of N active antenna elements in the near vicinity of R passive scatterers, where $N + R = M$. Therefore, the total number of array elements, passive and active, is M. Each of the active and passive elements has a known location,(x_m, y_m), and known unimodal radiation characteristics described by its isolated radiation pattern [5]

$$f_m(\theta, \phi), \quad m = 1 \dots M \tag{7}$$

where m is the index of the mth element in the array of active and passive elements.

Viewed as an M-port network, such an array can be represented by its impedance/admittance matrix, $Y = Z^{-1}$, which depends on the array geometry, its physical composition and the effects of electromagnetic coupling between the elements of the array and the nearby environment. Both Z and Y are $M \times M$ symmetric matrices. We define Y' to be the $N \times M$ subset of Y that includes only the mutual admittance elements between the active elements and all other elements in the array. Mutual admittances involving only the passive array elements are not included in Y'. It will be shown that Y', together with knowledge of the locations of all the elements in the array — active and passive — determines the active element patterns of the array, and therefore, the intrinsic radiation characteristics of such an array. We further define Y'' to be the $Q \times 1$ vector of the non-redundant elements of the matrix Y', accounting for the underlying symmetry of the the matrix Y ($Y_{mn} = Y_{nm}$). The number of elements in vector Y'' can be seen from simple enumeration to be $Q = MN - N(N-1)/2$.
Y is an $(M \times M)$ matrix:
Y' is an $(N \times M)$ subset of Y:
Y'' is a $(Q \times 1)$ vector:

$$Y = \begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1N} & \dots & Y_{1M} \\ Y_{21} & Y_{22} & \dots & Y_{2N} & \dots & Y_{2M} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ Y_{N1} & Y_{N2} & \dots & Y_{NN} & \dots & Y_{NM} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ Y_{M1} & Y_{M2} & \dots & Y_{MN} & \dots & Y_{MM} \end{bmatrix}$$

$$Y' = \begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1N} & \dots & Y_{1M} \\ Y_{21} & Y_{22} & \dots & Y_{2N} & \dots & Y_{2M} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ Y_{N1} & Y_{N2} & \dots & Y_{NN} & \dots & Y_{NM} \end{bmatrix}$$

$$Y''^T = \begin{bmatrix} Y_{11} & Y_{12} & \dots Y_{1M} & Y_{22} & \dots & Y_{2M} & \dots & Y_{NM} \end{bmatrix}$$

where Y''^T indicates the matrix transpose operation.
In the following, a method is derived to

1) extract the values of the elements of Y' and Y'' from measured or computed active element pattern data, and
2) form a linear approximation of Y'' as a function of variable element locations.

The admittance matrix and the linear approximation are used to account for the effects of mutual coupling on array performance while optimizing the location of the active and/or passive elements that comprise the array. The scan impedance at each of the actively excited antenna ports is also obtained. The procedure employs discrete inner product formulations and discrete processing.

6.2 Extraction of the Elements of Y' and Y''

The equivalent circuit for the nth port in this network is shown in Fig.1. Kirchhoff's voltage law equation for this circuit is given by

$$V_n = Z_{0n} \cdot I_n + \sum_{m=1}^{M} Z_{nm} \cdot I_m \qquad n = 1 \ldots N \tag{8}$$

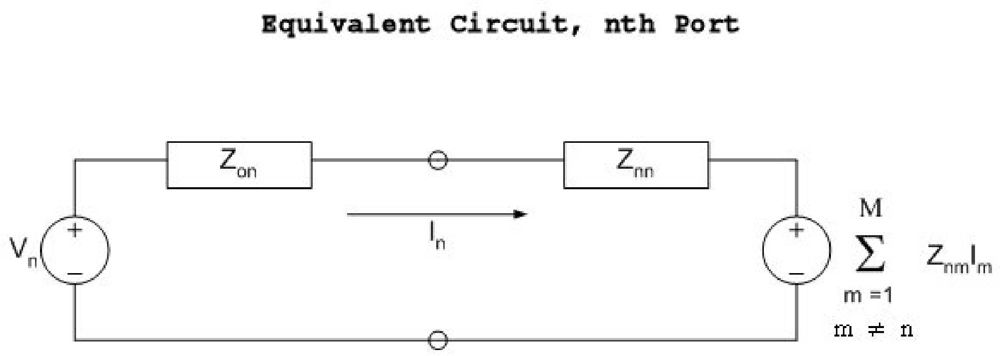

Fig. 1. Equivalent Circuit for the nth port

where Z_{nn} is the self-impedance of the nth element, Z_{0n} is the internal impedance of the nth port voltage source and Z_{nm} is the mutual impedance between the mth and nth ports.

The relationship between an arbitrary vector of excitation voltages, $\mathbf{V}$, impressed at the N active antenna terminals and the resulting set of currents, $\mathbf{I}$, induced in the M active and passive elements is given by $\mathbf{I} = Y\mathbf{V}$.

Now consider the case of exciting the nth active element with a unit voltage with all other elements terminated in their internal impedance. The passive elements can be assigned an internal impedance of zero. This form of excitation can be represented by the vector $\mathbf{V}\delta_n = \{\delta_{mn}\}$ where

$$\delta_{mn} = \begin{cases} 0, & \text{for } m \neq n \\ 1, & \text{for } m = n \end{cases}$$

The corresponding vector of induced currents is

$$\mathbf{I}\delta_n = Y\mathbf{V}\delta_n = (Y_{1n}, Y_{2n}, \ldots, Y_{Mn})^T \tag{9}$$

The nth active element pattern, excited by $\mathbf{V}\delta_n$, is given by:

$$h_n(\theta, \phi) = \sum_{m=1}^{M} Y_{nm} f_m(\theta, \phi) e^{j\frac{2\pi}{\lambda}\mathbf{r}_m \cdot \hat{\mathbf{u}}(\theta,\phi)},$$

$$n = 1 \ldots N \tag{10}$$

where $h_n(\theta, \phi)$ is a continuous function of (θ, ϕ). Only those elements of Y that enter into the set of N equations in (10) affect the active element patterns, $h_n(\theta, \phi)$. For simplicity, we set the isolated pattern functions $f_m(\theta, \phi) = 1$ and set $\theta = \pi/2$.

The set of element pattern data can be sampled in the ϕ-plane, such that $\phi_t = 2\pi t/T$, where t is the index of samples and $t = 0 \ldots T$. The minimum number of samples, T, is given as a rule of thumb by $T > 6\pi L/\lambda$, which is equal to 2π divided by $1/3$ the nominal array beamwidth, λ/L, where L is the maximum dimension of the array in meters. The expression for the sampled element patterns is given by:

$$h_{tn} = \sum_{m=1}^{M} Y_{nm} e^{jk(x_m \cos(\phi_t) + y_m \sin(\phi_t))} \qquad n = 1 \ldots N \tag{11}$$

where $k = 2\pi/\lambda$ and (x_m, y_m) are the (x, y) coordinates in meters of the mth element, active or passive, in the array. Let

$$ex_{tm} = e^{jk(x_m \cos(\phi_t) + y_m \sin(\phi_t))} \qquad m = 1 \ldots M \tag{12}$$

The ex_{tm} form a set of M linearly independent complex valued $(T \times 1)$ vectors. An orthonormal set, gs_{tn}, can be formed from the set ex_{tm} by application of the Gram-Schmidt procedure [13]. Let matrix $ex = \{ex_{tm}\}$ where $ex^{<m>} = mth$ column of ex; matrix $gs = \{gs_{tm}\}$ where $gs^{<m>} = mth$ column of gs; and matrix $h = \{h_{tn}\}$ where $h^{<n>} = nth$ column of h. In what follows we shall use the bra-ket notation of Dirac, in which $\langle x|y\rangle$ denotes the inner product of vectors x and y; and $|x\rangle\langle y|$ denotes their outer product.

Therefore $\langle ex^{<m>}|gs^{<n>}\rangle$ denotes the inner product of these two vectors. From the properties of the Gram-Schmidt construction, we have that:

$$\langle ex^{<m>}|gs^{<n>}\rangle = 0 \quad if \quad n > m \tag{13}$$

In this notation, eq.(11) becomes

$$h^{<n>} = \sum_{m=1}^{M} Y_{nm} ex^{<m>} \qquad n = 1 \ldots N \tag{14}$$

From (12) and (13) we have that

$$\langle h^{<0>}|gs^{<M>}\rangle = Y_{0M}\langle ex^{<M>}|gs^{<M>}\rangle, \quad \text{giving}$$

$$Y_{0M} = \frac{\langle h^{<0>}|gs^{<M>}\rangle}{\langle ex^{<M>}|gs^{<M>}\rangle} \tag{15}$$

Next we have that

$$\langle h^{<0>}|gs^{<M-1>}\rangle = Y_{0M-1}\langle ex^{<M-1>}|gs^{<M-1>}\rangle$$
$$+Y_{0M}\langle ex^{<M>}|gs^{<M-1>}\rangle, \quad \text{giving}$$

$$Y_{0M-1} = \frac{\langle h^{<0>}|gs^{<M-1>}\rangle - Y_{0M}\langle ex^{<M>}|gs^{<M-1>}\rangle}{\langle ex^{<M-1>}|gs^{<M-1>}\rangle} \tag{16}$$

Proceeding in this manner, using back substitution, all of the elements of Y' and Y'' that occur in the expressions for the N active element patterns of the array are obtained. The procedure just outlined for obtaining Y' and Y'' is readily implemented using digital processing. If necessary, re-ordering of the Gram-Schmidt orthogonalization can be used to eliminate round-off error.

6.3 Linear Approximation of Y'' as a Function of Variable Element Locations

The elements of $Y'', Y',$ and Y are each functions of the variable element locations of the array, but they are not functions of the externally impressed array excitations. A linear approximation of $Y''(\mathbf{v})$ in bra-ket notation has the following form:

$$Y''(\mathbf{v}) \approx Y''(\mathbf{v}^I) + J(\mathbf{v})|\mathbf{v} - \mathbf{v}^I\rangle \tag{17}$$

where $\mathbf{v}^I = \mathbf{v}$ at the Ith iteration of the optimization process, and the Jacobean matrix $J(\mathbf{v}) = |\partial Y''_{mn}(\mathbf{v})/\partial v_n|, \quad m = 1 \ldots Q$ and $n = 1 \ldots NV$. (Note, that $\mathbf{v}$ is the vector of variable array parameters, v_n is an element of that vector, and $\mathbf{V}$ is the vector of applied voltage excitations, and V_n is an element of that vector.)

Let J^{I-1} be the estimate of $J(\mathbf{v})$ at the $I-1$th iteration. Then a Broyden update to this estimate [16], such that $J^I = J^{I-1} + \Delta J^I$ satisfies (17) is given by

$$\Delta J^I = \frac{|\Delta Y''^I - J^{I-1}\Delta \mathbf{v}^I\rangle\langle\Delta \mathbf{v}^I|}{\langle\Delta \mathbf{v}^I|\Delta \mathbf{v}^I\rangle} \tag{18}$$

where $Y''^I = Y''(\mathbf{v}^I), \Delta\mathbf{v}^I = \mathbf{v}^I - \mathbf{v}^{I-1}, \Delta Y''^I = Y''^I - Y''^{I-1}$. At the first iteration, I=1, it is necessary to initialize the first estimate J^1. If no analytic estimates of the partial derivatives, $\partial Y''_{mn}(\mathbf{v})/\partial v_n$, are available; then finite difference estimates of the partial derivatives can be used. The empirical optimization algorithm, v2, incorporating the procedures described in Sections 6.1-3, is given in Sidebar 2.

SIDEBAR 2 - THE EMPIRICAL OPTIMIZATION ALGORITHM, V2

1) At the Ith iteration of the process, set array parameter values: $\mathbf{r}^I, \mathbf{a}^I, \mathbf{v}^I$
2) Measure or compute $P(\phi', \mathbf{v}^I)$
3) Evaluate $f(\mathbf{v}^I) = \|P(\phi', \mathbf{v}^I) - P_D(\phi')\|$
   If $f(\mathbf{v}^I) \le \varepsilon,$ STOP, $\varepsilon =$convergence criterion; Otherwise continue.
4) Measure the active element patterns, $h_n(\phi', \mathbf{r}^I), \quad n = 1 \ldots N$
   Sample the element patterns in the ϕ'-plane, ϕ'_t, and form N discrete $(T \times 1)$ vectors: $h_{tn} = h^{\langle n\rangle}, \quad n = 1 \ldots N$
5) Extract the admittance vector, Y''^I, from the $h^{\langle n\rangle}$ data:
   a) Set $ex^{\langle m\rangle} = e^{jk(x_m \cos(\phi_t) + y_m \sin(\phi_t))}$
   b) Use Gram-Schmidt procedure to transform vectors $ex^{\langle m\rangle}$ to orthonormal vectors $gs^{\langle m\rangle}$
   c) Find Y''^I by sequential inner products $\langle h^{\langle n\rangle}|gs^{\langle m\rangle}\rangle$ and 'back substitution'
      (If computation rather than measurement is used, Y''^I may be computed directly, without the need for steps 5.a-5.c. The $h^{\langle n\rangle}$ can then be obtained from eq.(14)
6) Form linear approximation $Y''(\mathbf{v})$:
   a) Set $\Delta\mathbf{v}^I = \mathbf{v}^I - \mathbf{v}^{I-1}, \quad \Delta Y''^I = Y''^I - Y''^{I-1}$ and

$$\Delta J^I = \frac{|\Delta Y''^I - J^{I-1}\Delta \mathbf{v}^I\rangle\langle\Delta \mathbf{v}^I|}{\langle\Delta \mathbf{v}^I|\Delta \mathbf{v}^I\rangle}$$

$$J^I = J^{I-1} + \Delta J^I$$

   b) Set $Y''(\mathbf{v}) = Y''^I + J^I|\mathbf{v} - \mathbf{v}^I\rangle$
7) Compute
   a)

$$h(\mathbf{v})_{tn} = \sum_{m=1}^{M} Y(\mathbf{v})_{nm} e^{jk(x_m \cos(\phi_t) + y_m \sin(\phi_t))}$$

   $n = 1 \ldots N$

b)

$$E(\mathbf{v})_t = \sum_{n=1}^{N} h(\mathbf{v})_{tn} e^{jk(x_n \cos(\phi_t) + y_n \sin(\phi_t))}$$

c)

$$P(\mathbf{v})_t = 20 \log \left(\left| \frac{E(\mathbf{v})_t}{E(\mathbf{v})_{t_0}} \right| \right)$$

normalized at angle sample t_0

(partial derivatives, $\partial P(\mathbf{v})_t / \partial V_n$, can also be obtained if required)

8) Set $f(\mathbf{v}) = \|P(\phi', \mathbf{v}) - P_D(\phi')\|, \mathbf{v}$ variable and, using iterative numerical search, find $\min f(\mathbf{v}) \longrightarrow \mathbf{v}^*$
9) Set $I = I + 1, \quad \mathbf{v}^I = \mathbf{v}^*$, go to step 1, and repeat

END Sidebar 2

6.4 Scan Admittance/Impedance

The relationship between an arbitrary vector of excitation voltages, $\mathbf{V}$, impressed at the N active antenna terminals and the resulting vector of currents, $\mathbf{I}$, induced in the N active elements is given by $\mathbf{I} = [Y'\ ]\mathbf{V}$, where $[Y']$ is the (NxN) subset of matrix Y' defined in Section 6.1. The corresponding system of equations is given by

$$\begin{aligned} I_1 &= Y_{11}V_1 + Y_{12}V_2 + \ldots + Y_{1N}V_N \\ I_2 &= Y_{21}V_1 + Y_{22}V_2 + \ldots + Y_{2N}V_N \\ &\vdots \\ I_N &= Y_{N1}V_1 + Y_{N2}V_2 + \ldots + Y_{NN}V_N \end{aligned} \tag{19}$$

The scan admittance, Ys_n, and scan impedance, Zs_n, at each of the actively excited antenna ports is therefore given by:

$$\begin{aligned} Ys_n &= \sum_{m=1}^{N} Y_{nm} \frac{V_m}{V_n} \\ Zs_n &= (Ys_n)^{-1}, \qquad n = 1 \ldots N \end{aligned} \tag{20}$$

7. Optimization Results

In this section, four examples of antenna array optimization using the empirical optimization algorithm are presented. These results were obtained via computer simulation; examples 1, 2 and 3 using version v1 of Section 2, and example 4 using version v2. Experimental results obtained with the empirical optimization algorithm using measured data are shown in references [11,12]. The method was found to be robust when tested using computer simulation of measurement round-off error. The computer program used to obtain the results shown in examples 1, 2 and 3 is available at www.huttsystems.com.

Examples 1, 2 and 4, deal with the case of linear arrays of parallel dipole (active) elements and passive wire scatterers. Coupling effects are computed using the induced emf method [1], which, for the case of parallel dipoles and thin wire scatterers, is reasonably accurate. Example 3 gives optimization results for an array of idealized uncoupled isotropic radiators, and illustrates the dramatic effectiveness of optimized non-uniform spacing in suppressing grating lobes in thinned arrays and in wide scan angle and broad frequency bandwidth array operation.

Example 1. In this example an array of 4 dipoles each of length 0.5λ and radius 0.002λ, spaced 0.8 wavelengths apart along the x-axis with a single passive scatterer located at $x = 1.6\lambda$ and $y = -0.25\lambda$ of length 0.6λ and radius 0.002λ is optimized according to a minimize max sidelobe level (SLL) criteria. Only the set of complex valued element excitations are optimized, $\mathbf{a}_n = |a_n|e^{j\alpha_n}$, element spacing is fixed at 0.8λ, and only one iteration is required to obtain a significant reduction in max SLL from -7.3 dB to -11.7 dB, see Fig.2. In this example the passive scatterer is offset from the line of symmetry. The scatterer is -0.25 wavelengths behind the 3rd element of the array. The excitation coefficients are initially set to a Chebyshev distribution for -30dB sidelobe levels.

The initial pattern (shown in red) has a maximum sidelobe level of -7.3dB. This high sidelobe level is due to the effects of mutual coupling. After searching in the space of complex excitation coefficients, an optimized pattern (shown in blue) is obtained with a maximum sidelobe level of -11.7dB. It is also noted that the initial pattern beamwidth exceeds the specified value due to coupling. The final pattern displays both lower sidelobes and a narrower beamwidth. The optimum set of element amplitudes (unit-less weights) and phases (radians) are shown in Table 1.

Table 1. Example 1 - Set of Optimized Element Amplitudes and Phases

n	1	2	3	4
$\|a_n\|$	1.065	0.997	1.024	1.093
α_n	-0.143	0.017	0.016	-0.156

Example 2. An array of 15 isotropic radiators, initially spaced 1λ apart, is optimized by searching for the set of non-uniform element locations to minimize its max SLL while maintaining uniform element excitation. The initial pattern has 0 dB max SLL due to the grating lobes at ± 90 degrees. Coupling effects are not included in view of the relatively large average inter-element spacing of 1λ. After 9 iterations, the optimization algorithm reduces the maximum sidelobe to -16dB as shown below in Fig.3. It is interesting to note that the optimized pattern has approximately equal sidelobe levels which is characteristic of a Chebyshev design, despite the fact that uniform element excitation has been maintained. The set of optimized set of element locations are shown in Table 2.

Table 2. Example - 2 Set of Optimized Element Locations

x_{1-4}	0.00	1.42	2.17	3.95
x_{5-8}	4.83	5.73	6.64	7.57
x_{9-12}	8.47	9.40	10.32	11.15
x_{13-15}	12.31	13.17	14.00	

Example 3. An array of 16 dipoles each of length 0.5λ and radius 0.002λ, with an average element spacing of 1λ, phase scanned to 30 degrees and a passive scatterer of length 0.6λ and radius 0.002λ, 0.75λ in front of the array midpoint, is to have its max SLL minimized. The initial array pattern, with uniform spacing and a 30 dB Chebyshev amplitude excitation distribution, has a 0 dB max SLL due to the appearance of a grating lobe. After 3 iterations of both element location and excitation optimization, the algorithm synthesizes an optimum set of non-uniform element locations and excitations giving a pattern with a max SLL of -11.5 dB. The optimum set of element amplitudes (unit-less weights) and phases (radians) are shown below in Table 3. Note that the grating lobe is effectively suppressed. The results are shown in Fig.4.

Table 3. Example 3 - Set of Optimized Element Locations, Amplitudes and Phases

x_n	$\|a_n\|$	α_n
0	0.84	0.64
1.49	1.10	0.92
2.53	1.25	0.79
3.37	1.50	0.83
4.12	1.71	0.89
5.45	1.75	0.67
6.62	2.42	0.64
7.05	2.17	0.80
8.63	1.49	0.64
9.26	1.85	0.60
10.31	1.72	0.92
11.43	2.01	1.08
11.98	1.42	1.19
12.68	1.37	0.68
14.69	1.58	0.46
15.00	1.16	0.72

Example 4. To illustrate the special features of algorithm version v2 described in Section 6, we choose the problem of finding the optimum set of non-uniform element spacings for a six element Yagi-Uda antenna array that minimize its maximum sidelobe level. This is a variation on the problem of maximizing the gain of a Yagi-Uda array [3]. The array consists of a passive reflector (element 1), an active driven element (element 2), and four passive directors (elements 3-6). The fixed element lengths are: $l1 = 0.472$, $l2 = 0.45$, $l3 = l4 = l5 = l6 = 0.438$; and each element radius = 0.0025, all dimensions are in wavelengths. The elements are spaced along the x-axis, their lengths parallel to the z-axis, with initial location coordinates $x_i = (-0.25, 0, 0.31, 0.62, 0.93, 1.24)$, and $y_i = 0, i = 1..6$. The initial azimuth (ϕ-plane) pattern, shown in Fig.5, has a max sidelobe level of -6.9 dB.

The values of the self and mutual admittances between element 2 and all other elements, which is equivalent to the value of the currents at each element terminal, is calculated (see Section 6.2) to be

$$Y_{2i} = \begin{bmatrix} -0.021 + j5.81 \times 10^{-3}, 0.032 + j3.91 \times 10^{-3}, -0.013 - j8.42 \times 10^{-3}, \\ -5.52 \times 10^{-3} + j0.015, 0.018 - j0.011, -0.017 - j7.68 \times 10^{-4} \end{bmatrix}$$

The second element of Y_{2i} above is the self admittance of the driven element. For the present case of a Yagi-Uda array, the scan impedance described in Section 6.5 is simply the input impedance and is equal to the reciprocal of the self admittance, i.e. $Zin = 30.9 - j3.8$ ohms.

After three iterations, the optimum set of x coordinates is found to be $x_i = (-.257, 0.03, 0.28, 0.59, 0.96, 1.27)$. The resulting azimuth pattern, shown in Fig.6, has a max sidelobe of -8.9 dB, a reduction of 2 dB.

The corresponding set of admittances is calculated to be

$$Y_{2i} = \begin{bmatrix} -9.09 \times 10^{-3} + j4.23 \times 10^{-3}, 0.017 + j7.478 \times 10^{-3}, -3.56 \times 10^{-3} - j0.012, \\ -7.4 \times 10^{-3} + j0.014, 0.013 - j9.33 \times 10^{-3}, -0.012 + j5.87 \times 10{-}4 \end{bmatrix}$$

giving $Zin = 48.9 - j21.3$ ohms. The gain for both the initial and final Yagi-Uda array is approximately 11.2 dB. These results were obtained by computer simulation using the induced EMF method.

8. Conclusion

Empirical optimization is an experimental-computational algorithm for the optimization of antenna array performance accounting for the effects of mutual coupling and scattering between the elements of the array and the nearby environment. The algorithm is applicable to arrays of various element types having arbitrary configurations, including phased arrays, conformal arrays and non-uniformly spaced arrays. The algorithm can find both the optimum set of array element locations (non-uniform spacing) as well as the optimum set of element excitations. This provides added degrees of freedom in achieving optimum array performance and in compensating for coupling effects, as compared to traditional analytical design methods. Non-uniform spacing offers special advantages in suppressing grating lobes in thinned arrays and in wide angle scan and broad frequency bandwidth array operation.

Two versions of the algorithm have been presented. Both deal with arrays having both active and passive elements. The first, version v1, is applicable to the optimization of the locations and excitations of the active elements in an array, but not the passive elements. The second, version v2, is an extension of the first, and is applicable to the optimization of the locations of all elements in an array, active and passive, and to excitation optimization of the active array elements. In version v2, a novel method was presented in which the admittance matrix representing an antenna array consisting of both active and passive elements, is extracted from the array's element pattern data. The admittance matrix includes the effects of electromagnetic coupling between the active and passive elements of the array. The method presented for extraction of the admittance matrix is applicable to arbitrary array configurations including arrays with parasitic elements and conformal and non-uniformly spaced arrays. Other methods to account for coupling effects in arrays described in the literature normally require uniform element spacing and do not account for parasitic elements. It was shown that element scan impedances can be obtained from the matrix product of a subset of the admittance matrix with an arbitrary element excitation vector. The admittance matrix formulation incorporated into the empirical optimization algorithm provides an efficient method to find the optimum set of non-uniform element spacings for array performance optimization.

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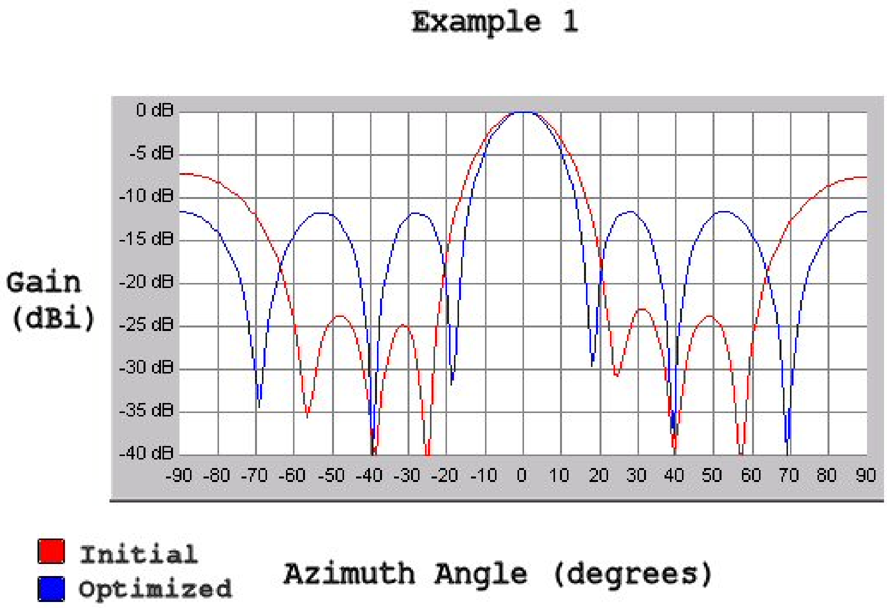

Fig. 2. Example 1

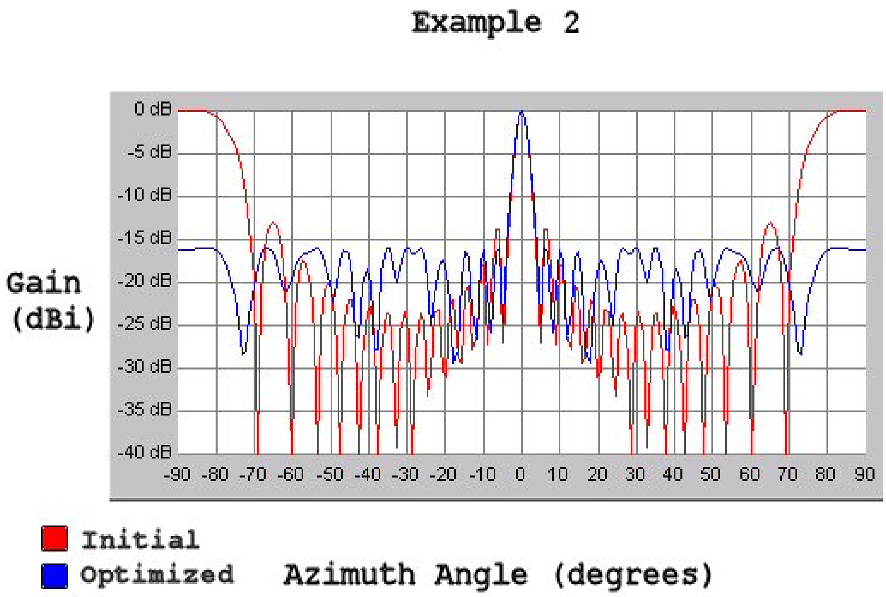

Fig. 3. Example 2

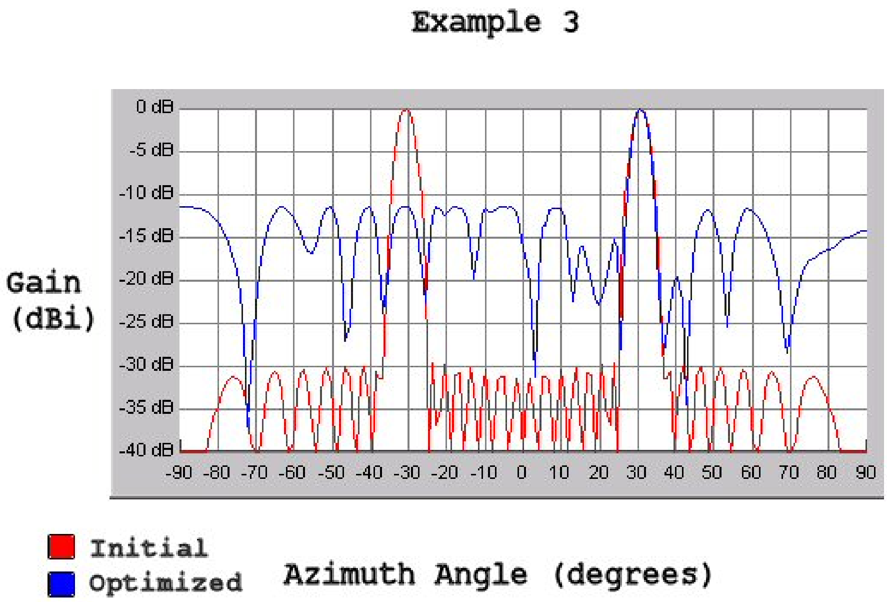

Fig. 4. Example 3

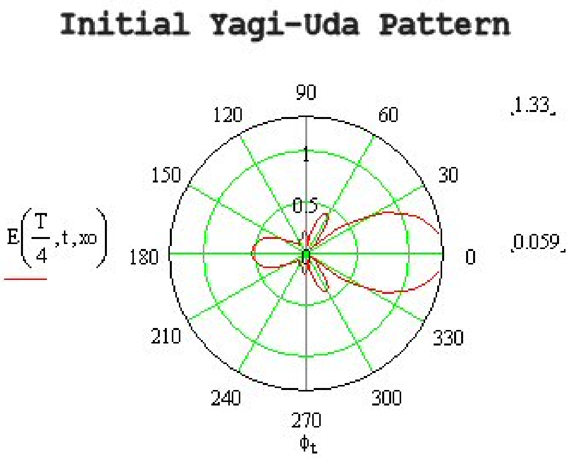

Fig. 5. Example 4 - Initial Yagi-Uda Pattern

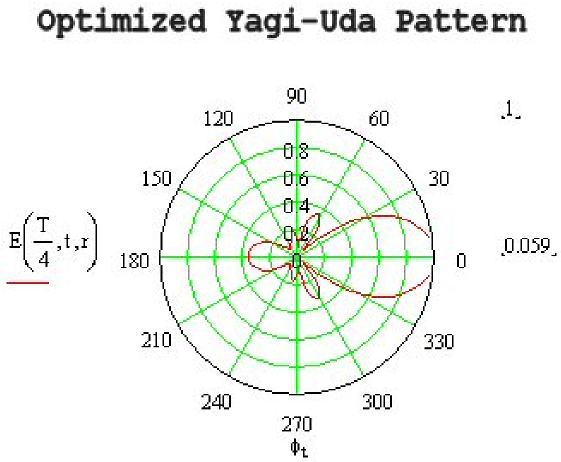

Fig. 6. Example 4 - Optimized Yagi-Uda Pattern